Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **October/November 2020**

3 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document has **8** pages. Blank pages are indicated.

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Section A: Probability (60 marks)

1 The text of a printed book may be considered as a series of individual symbols. It may be assumed that each symbol is equally likely to be misprinted, independently of any other symbol. The mean number of misprints per 5000 symbols is 0.4.

(a) State the exact distribution of the number of misprints per 5000 symbols. [2]

(b) Explain why this distribution can be well approximated by the distribution Po(0.4). [1]

(c) It is given that n is the largest number of symbols for which the probability that all are printed correctly is greater than 0.99. Use a Poisson distribution to estimate the value of n. [4]

2 The continuous random variables X and Y have independent distributions $\mathrm{N}(3\mu,\ 7\sigma^2)$ and $\mathrm{N}(4\mu,\ 16\sigma^2)$ respectively, where $\mu \neq 0$ and $\sigma \neq 0$. The random variable Z, given by $Z = \alpha X + \beta Y$, where α and β are constants, is an unbiased estimator of μ.

(a) Find a relationship between α and β. [2]

(b) Find the values of α and β for which $\mathrm{Var}(Z)$ is as small as possible, and state this smallest possible value of $\mathrm{Var}(Z)$ in terms of σ^2. [5]

3 The random variable X has the distribution $\mathrm{Po}(\lambda)$, where λ is a positive integer.

(a) Write down an expression for $\mathrm{P}(X = \lambda)$. [1]

(b) For the case $\lambda = 60$, use a normal approximation to find $\mathrm{P}(X = 60)$. [4]

(c) It is given that, for any random variable Y with the distribution $\mathrm{N}(\mu,\ \sigma^2)$,

$$\mathrm{P}(y - 0.5 \leqslant Y \leqslant y + 0.5) \approx \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2},$$

provided μ and σ are large.

Use this result and your answer to part **(a)** to show that, if λ is a large positive integer, then $\lambda! \approx \sqrt{2\pi\lambda}\,\mathrm{e}^{-\lambda}\lambda^{\lambda}$. [2]

(d) Hence find an approximation to $\ln(75!)$, giving your answer correct to 7 significant figures. [2]

4 **(a)** The discrete random variable R has the distribution $\mathrm{Geo}(p)$. Show that the probability generating function of R is $\dfrac{pt}{1 - qt}$, where $q = 1 - p$. [4]

(b) A bag contains 30 red balls and 70 blue balls. A ball is chosen at random from the bag, its colour is noted, and it is then replaced. This process is repeated. The Nth ball chosen is the 5th red ball to be chosen. Use probability generating functions to find $\mathrm{P}(N = 8)$. [5]

5 A geographer collected data about the mid-day temperature in London (x °C) on 15 randomly chosen autumn days and in Paris (y °C) on 17 randomly chosen autumn days. The results are summarised as follows.

London	Mean 11.6	Standard deviation 5.6
Paris	$\Sigma y = 243$	$\Sigma y^2 = 4070$

(a) Stating a necessary assumption, calculate a 99% confidence interval for the mean mid-day temperature on autumn days in London. Give the end-points of the interval correct to 2 decimal places. [5]

(b) Stating any further necessary assumptions, calculate a 95% confidence interval for the difference in the means of the mid-day temperatures on autumn days in London and Paris. Give the end-points of the interval correct to 2 decimal places. [8]

6 The continuous random variable S has probability density function

$$\mathrm{f}(x) = \begin{cases} \frac{2}{9}x(3-x) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The continuous random variable T has probability density function

$$\mathrm{g}(x) = \begin{cases} k & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Sketch on the same axes the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. [3]

(b) Without carrying out any calculations, state with a reason which of S and T has the larger variance. [2]

(c) The number u satisfies $\mathrm{P}(S < u) = 0.75$. Verify that u lies between 2.02 and 2.03. [5]

(d) The random variable Y is defined by $Y = \sqrt{S}$. Find the cumulative distribution function of Y. [5]

Section B: Mechanics (60 marks)

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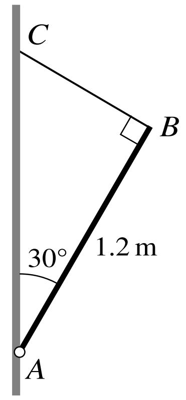

A non-uniform rod AB of mass 2 kg and of length 1.2 m is smoothly hinged at A in a vertical plane perpendicular to a vertical wall. The rod is kept in equilibrium by a light inextensible string attached to B and to the wall at C, vertically above A, where angle $ABC = 90°$ and angle $CAB = 30°$ (see diagram). The tension in the string is 7.5 N.

(a) Find the distance of the centre of mass of the rod from A. [2]

(b) Find the horizontal and vertical components of the force exerted by the hinge on the rod at A. [4]

8 A box of mass 20 kg is pulled down a rough plane inclined at 15° to the horizontal. The pulling force acts parallel to a line of greatest slope of the plane. The box starts from rest and after having moved a distance of 10 m its speed is $3\,\text{m}\,\text{s}^{-1}$. The coefficient of friction between the box and the plane is 0.8. Calculate the work done by the pulling force on the box. [8]

9 A particle S is fixed to a horizontal turntable at a point 0.1 m from the centre. The turntable rotates in such a way that its angular speed $\omega\,\text{rad}\,\text{s}^{-1}$ at time t s is given by $\omega = 2 + \sin 2t$. When $t = \frac{1}{6}\pi$ find

(a) the speed of S, [2]

(b) the magnitude of the acceleration of S. [6]

10 A particle P of mass 0.6 kg is fixed to one end of a light inextensible string of length 0.2 m. The other end of the string is fixed at the point O. P is held with the string taut and OP horizontal, and P is then projected downwards with speed $1.4\,\text{m}\,\text{s}^{-1}$ so that it begins to move in a vertical circle.

(a) Find the tension in the string when P is at the lowest point of its path. [4]

The string becomes slack when P reaches a point Q.

(b) Find the magnitude and direction of the velocity of P at the point Q. [6]

(c) Find the greatest height above Q that is reached by P. [3]

11 A drone is programmed to move in a straight line so that at time $t = 0$ it is at the origin and is moving with a velocity of $4\,\mathrm{m\,s^{-1}}$ in the positive x-direction. When its velocity is $v\,\mathrm{m\,s^{-1}}$, its acceleration is $0.1v(12 - v)\,\mathrm{m\,s^{-2}}$.

(a) By setting up and solving a differential equation, show that the velocity is given by

$$v = \frac{12e^{1.2t}}{e^{1.2t} + 2},$$

where t is the time in seconds. [8]

(b) Write down the limiting velocity of the drone as t becomes large. [1]

(c) At time t s, the displacement of the drone from the origin is x m. Find an expression for x in terms of t. [4]

12 A particle A of mass 0.2 kg is suspended from a ceiling by a light elastic string of natural length 0.5 m and modulus of elasticity 2.5 N. A hangs in equilibrium.

(a) Find the extension of the string. [2]

A small ring B of mass 0.05 kg is threaded on the string and projected vertically downwards so that it collides with A. Immediately before the collision, the speed of B is $5.3\,\mathrm{m\,s^{-1}}$, and as a result of the collision A and B coalesce.

(b) Find the common speed of A and B immediately after the collision. [2]

(c) Show that, after the collision, A and B together perform simple harmonic motion. (It may be assumed that the string does not become slack.) [5]

(d) Find the amplitude of the simple harmonic motion. [3]

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